

Arraying Techniques for the Galileo S-Band Mission, Part I- Full Spectrum Combining *

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Abstract

It has been known for some time now that when signals from a single source are received at multiple antennas, these antennas can be arrayed to enhance the quality of the received signal. What has been relatively unknown, however, is that the quality of the combined signal critically depends on the arraying algorithm, as well as, the characteristics of the transmitted signal. This two part article describes two arraying techniques known as Full Spectrum Combining (FSC) and Complex Symbol Combining (CSC), and then compares them using signals whose characteristics represent those from the Galileo spacecraft using its low gain antenna (at S-band) at the time it reaches Jupiter. Various combinations of existing deep space antennas are used to compare these techniques. This part of the article describes the FSC algorithm, and then analyzes its performance. In the process, two measures of signal quality, symbol signal-to-noise ratio (SNR) degradation and symbol SNR loss, are defined and their application discussed.

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1 Introduction

In deep space communications, combining signals from multiple antennas is commonly referred to as arraying. Arraying techniques are important because they can significantly enhance system performance. For example, if signal power-to-noise density ratio (P/N_0) is a measure of system performance, then the effective P/N_0 after arraying should be ideally equal to the sum of the P/N 's corresponding to individual antennas. A typical arraying design trades complexity and gain (or improvement in system performance). Arraying is an attractive option for communication links operating near threshold. For instance, consider the Galileo spacecraft which is currently on its way to Jupiter. Due to a malfunctioned high gain antenna, Galileo must rely on its low gain S-band (2.2 to 2.3 GHz) antenna, and a much reduced link margin, for data transmission to earth. The Galileo S-band mission will employ arraying, as well as other techniques such as suppressed carriers and data compression, to improve its link margin and maximize data return. The current plan is to implement an inter-continental array between 70-111 and 34-111 antennas at complexes in Australia, Spain, and the United States. Specifically, this article considers the following antenna combinations: two 70-m antennas; one 70-m and one 34-m antenna; one 70-111 and two 34-m antennas; one 70-m and three 34-111 antennas; and four 34-m antennas. Even when communication links are operating above threshold, arraying is an economically attractive means of increasing the scientific return of a mission without having to build larger more expensive antennas. Smaller, inexpensive antennas can be built at less cost than a single large antenna, but with at least an equivalent performance after proper arraying.

Parts one and two of this article describe two antenna-arraying techniques for the Galileo S-band mission. The first, Full-Spectrum Combining (FSC), was chosen based on a previous study [1] that compared several antenna-arraying techniques, and showed that FSC resulted in the least degradation for weak signals. The second, which hasn't been considered before, is the Complex Symbol Combining (CSC) technique which is a viable alternative because of

the recent use of all-digital receivers in NASA's deep space communications network [2]. The FSC technique discussed in part I of this article is depicted in Fig. 1 (a). As shown, in FSC the received radio frequency (RF) signal at each antenna is down converted to an intermediate frequency (IF), transmitted to a central location where it is aligned and combined with signals from other antennas, and then demodulated by a single receiver chain. The chain consists of one carrier loop, one subcarrier loop, one symbol synchronization loop, and one matched filter. The RF/IF downconverter is assumed to output a complex IF signal (two IF signals that are orthogonal) denoted by the double lines in Fig. 1(a). The processing needed to align and combine the IF signals is shown in Fig. 1(L) for an array of two-antennas. The details of this scheme are discussed in the section on FSC performance. The CSC technique is discussed in part II of this article.

Part I has two objectives: the first is to briefly describe the performance of FSC in terms of SNR degradation, the measure of signal quality used in [1]; the second is to extend the results of [1] by evaluating the FSC performance in terms of SNR loss, which is always a more precise measure of signal quality. Symbol SNR degradation is defined as the ratio of the SNR at the matched filter output in the presence of non-ideal synchronization to the SNR in the presence of ideal synchronization. On the other hand, symbol SNR loss is defined as the additional symbol SNR needed in the presence of imperfect synchronization to achieve the same symbol error rate (SER) as in the presence of perfect synchronization. Mathematical representations of degradation and loss are given in the next section. Comparatively, the loss gives the absolute performance while degradation gives the relative performance advantage of an arraying scheme. Moreover, since the calculation of degradation is less demanding than computation of loss [3], it is the preferred calculation method at low symbol SNRs where it is approximately equal to loss. In the next section, the degradation and loss for a single antenna are derived; results from this section are, consequently, used to derive the degradation and loss for FSC, and illustrated via various numerical examples.

2 Single Receiver Performance

in deep space communications, the downlink symbols are first modulated onto a square-wave subcarrier; the modulated subcarrier then modulates an RF carrier [4]. This allows transmission of a residual carrier component whose frequency doesn't coincide with the data spectrum. At the receiver, the deep-space signal is demodulated using a carrier-tracking loop, a subcarrier-tracking loop [5], and a symbol-synchronizer loop [6] as shown in Fig. 2. Depending on the modulation index, carrier tracking can be achieved by a phase-locked loop (PLL), Costas loop, or both [7]. The PLL or a combination of loops is used for modulation indices less than 90 degrees whereas a Costas loop is used when the modulation index is equal to 90 degrees. The deep space signal with the carrier fully suppressed¹ can be represented as [8]

$$r(t) = \sqrt{2P}d(t)\text{Sqr}(\omega_{sc}t - \theta_{sc}) \cos(\omega_c t - \theta_c) + n(t) \quad (1)$$

where P is the received data power in Watts (W); ω_c and θ_c are the carrier angular frequency in radians per second (rads/sec) and phase in rads, respectively; and $\text{Sqr}(\omega_{sc}t - \theta_{sc})$ is the square-wave subcarrier with subcarrier angular frequency ω_{sc} in rads/sec and subcarrier phase θ_{sc} in rads. The symbol stream, $d(t)$, is given by

$$d(t) = \sum_{k=-\infty}^{\infty} d_k p(t - kT) \quad (2)$$

where d_k is the ± 1 binary data for the k^{th} symbol and T is the symbol period in seconds. The baseband pulse, $p(t)$, is unit power and limited to T seconds. The narrow-band noise $n(t)$ can be written as

$$n(t) = \sqrt{2}n_c(t)\cos(\omega_c t - \theta_c) - \sqrt{2}n_s(t)\sin(\omega_c t + \theta_c) \quad (3)$$

where $n_c(t)$ and $n_s(t)$ are statistically independent, stationary, band-limited, white Gaussian noise processes with one-sided spectral density level N_0 (W/Hz) and one-sided bandwidth

¹ This article considers the Galileo S-Band scenario in which the carrier is fully suppressed.

$W_n(\text{Hz})$, which is large compared to $\frac{1}{T}$. After signal demodulation, the symbol stream at the output of the matched filter in Fig. 2 can be written as [8]

$$v_k = \begin{cases} \sqrt{P}C_cC_{sc}d_k + n_k & d_k = d_{k-1} \\ \sqrt{P}C_cC_{sc}(1 - \frac{|\phi_{sy}|}{\pi})d_k + n_k & d_k \neq d_{k-1} \end{cases} \quad (4)$$

where the noise n_k is a Gaussian random variable with variance $\sigma_n^2 = \frac{N_0}{2T}$. The signal reduction functions C_c and C_{sc} are due to imperfect carrier and subcarrier synchronization and are given by [1]

$$C_c = \cos \phi_c \quad (5)$$

$$C_{sc} = 1 - \frac{2}{\pi}|\phi_{sc}| \quad (6)$$

where ϕ_c and ϕ_{sc} (in rads) respectively denote the carrier and subcarrier phase tracking errors. The symbol timing error, ϕ_{sy} , which affects the output only when there is a symbol transition, reduces the signal amplitude by $1 - \frac{|\phi_{sy}|}{\pi}$. Ideally, $\phi_c = \phi_{sc} = \phi_{sy} = 0$ and (4) reduce to the ideal matched filter output $v_k = \sqrt{P}d_k + n_k$, as expected. In writing (4), it is assumed that the carrier, subcarrier, and symbol loop bandwidths are much smaller than the symbol rate so that the phase errors ϕ_c , ϕ_{sc} , and ϕ_{sy} can be modeled as constant over several symbols. Throughout this article, ϕ_c is assumed to be Tikhonov distributed, and ϕ_{sc} and ϕ_{sy} are assumed to be Gaussian distributed. Let $p_c(\phi_c)$, $p_{sc}(\phi_{sc})$, and $p_{sy}(\phi_{sy})$ denote respectively the carrier, subcarrier, and symbol phase error density functions. Then²

$$p_c(\phi_c) = \begin{cases} \frac{\exp(\frac{1}{4}\rho_c \cos 2\phi_c)}{\pi I_0(\frac{1}{4}\rho_c)} & |\phi_c| \leq \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

where $I_k(x)$ denotes the modified Bessel function of order k , and ρ_c is the suppressed carrier or Costas loop SNR. Also, $p_{sc}(\phi_{sc})$ and $p_{sy}(\phi_{sy})$ are given by

$$p_i(\phi_i) = \frac{\exp(-\phi_i^2/2\sigma_i^2)}{\sqrt{2\pi}\sigma_i}, \quad i = sc, sy \quad (8)$$

²It is assumed that the Costas loop locks at zero phase error. The π lock point can also be handled by an appropriate transformation.

where σ_{sc}^2 and σ_{sy}^2 are the reciprocals of the subcarrier and symbol loop SNRs, respectively, denoted as ρ_{sc} and ρ_{sy} . The carrier [8], subcarrier [8], and symbol [6] loop SNRs are respectively given as

$$\rho_c = \frac{P/N_0}{B_c} \left(1 + \frac{1}{2E_s/N_0}\right)^{-1} \quad (9)$$

$$\rho_{sc} = \left(\frac{2}{\pi}\right)^2 \frac{P/N_0}{W_{sc}B_{sc}} \left(1 + \frac{1}{2E_s/N_0}\right)^{-1} \quad (10)$$

$$\rho_{sy} = \frac{P/N_0}{2\pi^2 W_{sy}B_{sy}} \frac{\left(\text{erf}\left(\sqrt{\frac{E_s}{N_0}}\right) - \frac{W_{sy}}{2\sqrt{\pi}}\sqrt{\frac{E_s}{N_0}}\exp\left(-\frac{E_s}{N_0}\right)\right)^2}{\left(1 + \frac{E_s}{N_0}\frac{W_{sy}}{2} - \frac{W_{sy}}{2}\left[\frac{1}{\sqrt{\pi}}\exp\left(-\frac{E_s}{N_0}\right) - \sqrt{\frac{E_s}{N_0}}\text{erf}\left(\sqrt{\frac{E_s}{N_0}}\right)\right]^2\right)} \quad \text{“1”}$$

where $E_s/N_0 = PT/N$ is the symbol SNR, $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-v^2)dv$ is the error function, and B_c , B_{sc} , and B_{sy} (in Hz) denote the single-sided carrier, subcarrier, and symbol loop bandwidths, respectively. The parameters W_{sc} and W_{sy} , which denote the subcarrier and symbol window, are unitless and limited to (0, 1].

A useful quantity needed to compute degradation and loss is the symbol SNR conditioned on ϕ_c, ϕ_{sc} , and ϕ_{sy} . The conditional symbol SNR, denoted by SNR' , is defined as the square of the conditional mean of v_k divided by the conditional variance of v_k , i.e.,

$$SNR' = \frac{\overline{(v_k/\phi_c, \phi_{sc}, \phi_{sy})}^2}{\sigma_n^2} = \begin{cases} \frac{2PT}{N_0} C_c^2 C_{sc}^2 & d_k = d_{k-1} \\ \frac{2PT}{N_0} C_c^2 C_{sc}^2 \left(1 - \frac{|\phi_{sy}|}{\pi}\right)^2 & d_k \neq d_{k-1} \end{cases} \quad (12)$$

where $\overline{(z/y)}$ denotes the statistical expectation of x conditioned on y , and v_k and σ_n^2 are defined earlier.

2.1 Degradation

The symbol SNR degradation is defined as the ratio of the unconditional SNR at the output of the matched filter in the presence of imperfect synchronization to the ideal matched

filter output SNR. The unconditional SNR, denoted by SNR , is found by first averaging (12) over the symbol transition probability, and then over the carrier, subcarrier, and symbol phases. Letting \bar{x} denotes the average of x , the unconditional SNR is given as

$$SNR = \frac{2PT}{N_0} \overline{C_c^2} \overline{C_{sc}^2} \overline{C_{sy}^2} \quad (13)$$

where the signal amplitude reduction due to symbol timing errors (averaged over the symbol transition probability) is denoted C_{sy} , and given as

$$C_{sy} = 1 - \frac{|\phi_{sy}|}{2\pi} \quad (14)$$

Averaging over the phases yields []

$$\overline{C_c^2} = \frac{1}{2} \left[1 + \frac{I_1(\frac{1}{4}\rho_c)}{I_0(\frac{1}{4}\rho_c)} \right] \quad (15)$$

$$\overline{C_{sc}^2} = 1 - \sqrt{\frac{32}{\pi^3}} \frac{1}{\sqrt{\rho_{sc}}} + \frac{4}{\pi^2} \frac{1}{\rho_{sc}} \quad (16)$$

$$\overline{C_{sy}^2} = 1 - \sqrt{\frac{2}{\pi^3}} \frac{1}{\sqrt{\rho_{sy}}} + \frac{1}{4\pi^2} \frac{1}{\rho_{sy}} \quad (17)$$

Ideally, when there are no phase errors (i.e., when $\rho_c = \rho_{sc} = \rho_{sy} = 0$), $\overline{C_c^2} = \overline{C_{sc}^2} = \overline{C_{sy}^2} = 1$ and (13) reduce to $SNR_{ideal} = 2PT/N_0$, as expected. The degradation, D , for a single antenna is thus given by

$$D = -10 \log_{10} \left(\frac{SNR}{SNR_{ideal}} \right) = -10 \log_{10} \overline{C_c^2} \overline{C_{sc}^2} \overline{C_{sy}^2} \quad (18)$$

2.2 Loss

For the single receiver shown in Fig. 2, the SER, denoted $P_s(E)$, is defined as

$$P_s(E) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P'_s(E) p_c(\phi_c) p_{sc}(\phi_{sc}) p_{sy}(\phi_{sy}) d\phi_{sy} d\phi_{sc} d\phi_c = f \left(\sqrt{\frac{E_s}{N_0}} \right) \quad (19)$$

where $f(\cdot)$ is the functional relationship between SER and $\sqrt{\frac{E_s}{N_0}}$, and $P'_s(E)$ is the SER conditioned on the phase errors ϕ_c, ϕ_{sc} , and ϕ_{sy} . Following similar steps as in [9], the conditional SER can be shown to be

$$P'_s(E) = \frac{1}{4} \operatorname{erfc} \left(\sqrt{SNR'} \text{ when } d_k \neq d_{k-1} \right) + \frac{1}{4} \operatorname{erfc} \left(\sqrt{SNR'} \text{ when } d_k = d_{k-1} \right) \quad (20)$$

where

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-v^2) dv = 1 - \operatorname{erf}(x) \quad (21)$$

is the complementary error function. Substituting (12) for SNR' in (20) yields

$$P'_s(E) = \frac{1}{4} \operatorname{erfc} \left[\sqrt{\frac{E_s}{N_0}} C_c C_{sc} \left(1 - \frac{|\phi_{sy}|}{\pi} \right) \right] + \frac{1}{4} \operatorname{erfc} \left[\sqrt{\frac{E_s}{N_0}} C_c C_{sc} \right] \quad (22)$$

Ideally, when there are no timing errors (i.e., when $\rho_c = \rho_{sc} = \rho_{sy} = \text{cm}$), $C_c = C_{sc} = (1 - \frac{|\phi_{sy}|}{\pi}) = 1$ and (22) reduce to the well known binary phase shift keyed (BPSK) error rate, $P_s(E) = \frac{1}{2} \operatorname{erfc}(\sqrt{\frac{E_s}{N_0}})$.

Symbol SNR loss is defined as the additional symbol SNR needed in the presence of imperfect synchronization to achieve the same SER as in the presence of perfect synchronization. Mathematically, the SNR loss due to imperfect carrier, subcarrier, and symbol timing references is given in dB as

$$L_s = -20 \log \left[f^{-1}(P_s(E)) \right] \Big|_{[\text{infinite loop SNR}]} + 20 \log \left[f^{-1}(P_s(E)) \right] \Big|_{[\text{finite loop SNR}]} \quad (23)$$

where $P_s(E)$ is defined in (19). The first term in (23) is the value of E_s/N_0 required at a given value of $1/(N)$ in the presence of perfect synchronization whereas the second term is the value of E_s/N_0 required for imperfect synchronization. Note that loss defined in this way is a positive number.

3 FSC Performance

The FSC technique, depicted in Fig. 1(a), combines IF signals from multiple antennas and then demodulates the combined signal using the single receiver described in the previous

section. The resulting gain is maximized by aligning the IF signals in time and phase prior to demodulation [1]. The alignment algorithm for an array of two antennas is shown in Fig. 1 (b). Here signal 1 is assumed to be delayed by τ seconds, with respect to signal 2. The IF signal from antenna 2 is first delayed by $\hat{\tau}$ seconds, where $\hat{\tau}$ can be the output of the delay estimation loop, or it may be predicted from the geometric arrangement of the antennas and spacecraft. After delay compensation, both signals are input to the phase difference estimator which outputs $\hat{\theta}_{21}$, the estimate of θ_{21} , which is the phase of signal 2 relative to signal 1 at the estimator input. Subsequently, signal 2 is phase shifted by an amount equal to $-\hat{\theta}_{21}$, scaled by the weighting factor³ $\beta_2^A \sqrt{\frac{P_2}{P_1} \frac{N_{01}}{N_{02}}}$ [3], and then combined (or added) with signal 1. Notice in Fig. 1 (b) that the phase estimator filters the IF signal such that only the first L_{sc} harmonics of the IF spectrum are used for phase estimation. It is shown later that the accuracy of the estimates depends on B_{corr} , the bandwidth of the bandpass filter (BPF) centered at 1 IF; T_c , the estimation period; and L_{sc} , the number of subcarrier harmonics that pass the BPF unfiltered.

The symbol SNR degradation and loss analysis for FSC closely follows the analysis of the previous section as the combined signal is demodulated by a single receiver. As before, imperfect carrier, subcarrier, and symbol synchronization are expected to reduce the symbol SNR. In addition to those effects, however, the reduction due to imperfect combining must be accounted for as well. Assuming that the IF signals in Fig. 1 (b) are perfectly aligned in time ($\hat{\tau} = \tau$) but misaligned in phase⁴, the matched filter output for FSC is given by (4) with the modification that the one-sided noise power spectral density (PSD) level N_0 is now equal to the effective one-sided noise level $N_{0_{eff}}$ of the combined signal, and the data power P is now equal to the combined power P' conditioned on the phase alignment error. The

³In general, $\beta_n^A \triangleq \sqrt{\frac{P_n}{P_1} \frac{N_{01}}{N_{0n}}}$

⁴At low data rate such as the Galileo S-band mission, the timing alignment is less critical than the phase alignment. At high data rates, the timing alignment is comparable to the phase alignment.

effective one-sided noise PSD level at the matched filter input is given by [1]

$$N_{0_{eff}} = N_{01} \sum_{n=1}^L \gamma_n \quad (24)$$

where

$$\gamma_n \triangleq \frac{P_n N_{01}}{P_1 N_{0n}} \quad (25)$$

and where P_n and N_{0n} , denote respectively the signal power and one-sided noise PSD level of antenna n . Table 1 lists the γ_n factors⁵ for several DSN antennas at both S-band and X-band (8.4 to 8.5 GHz). Throughout this article, the ratio P_1/N_{01} is taken to be the signal power to one-sided noise PSD level of the reference antenna which, by convention, is taken to be the antenna with the highest gain. Consequently, in this article $\gamma_n \leq 1$. Table 1 lists the gamma values for 70- and 34-m antennas assuming the 70-m antenna is the reference antenna. The same table can be re-used for an arbitrary reference antenna as follows. Consider a three-element array consisting of one high-efficiency (11 dB) 34-m antenna and two standard (STD) 34-m antennas operating at X-band. Let the 34-m HRF be the reference antenna with $\gamma_1 = 1$, then the 34-m STD antennas have $\gamma_2 = \gamma_3 = 0.13/0.26 = 0.5$.

Let the phase alignment error between signal n and signal 1 be denoted by $\Delta\phi_{n1} = \theta_{n1} - \hat{\theta}_{n1}$, then the combined signal power conditioned on $\Delta\phi_{n1}$ is given as [1]

$$P' = P_1 \sum_{n=1}^L \sum_{m=1}^L \gamma_n \gamma_m C_{nm} \quad (26)$$

where

$$C_{nm} = e^{j(\Delta\phi_{n1} - \Delta\phi_{m1})} \quad (27)$$

is the complex signal reduction function due to phase misalignment. To summarize, the matched filter output of FSC is given by (4) after replacing P by P' as given by (26) and replacing N_0 by $N_{0_{eff}}$ which is given by (24).

⁵ *Deep Space Network/Flight Project Interface Design Handbook*, Document 810-5, Rev. D, vol. I (internal document), Jet Propulsion Laboratory, Pasadena, California, Modules TCI-10, TCI-30, and TLM-10, 1988.

A useful quantity needed in later calculations is $\overline{C_{nm}}$. In (27), assuming that the residual phase error for each antenna pair, $\Delta\phi_{n1}$ for $n = 1, \dots, L$, to be Gaussian distributed, with zero mean and variance $\sigma_{\Delta\phi_{n1}}^2$, and statistically independent from $\Delta\phi_{m1}$ for $n \neq m$, then it can be shown that [1]

$$\overline{C_{nm}} = \begin{cases} e^{-\frac{1}{2}(\sigma_{\Delta\phi_{n1}}^2 + \sigma_{\Delta\phi_{m1}}^2)} & n \neq m \\ 1 & n = m \end{cases} \quad (28)$$

where the variance of the residual phase error can be related to the SNR of the correlator as follows

$$\sigma_{\Delta\phi_{n1}}^2 = \frac{1}{2SNR_{n1, fsc}} \quad (29)$$

Here, $SNR_{n1, fsc}$ denotes correlator SNR [or SNR of the complex signal \tilde{x} in Fig. 1 (b)], and is shown in Appendix A to equal

$$SNR_{n1, fsc} = \frac{T_c \left(\frac{A}{\pi}\right)^2 \frac{P_1}{N_{01}} \left[\sum_{\substack{j=1 \\ j: \text{odd}}}^{L_{sc}} \frac{1}{j^2} \right]}{1 + \frac{1}{\gamma_n} + 2\left(\frac{\pi}{4}\right)^2 B_{corr} \sum_{\substack{j=1 \\ j: \text{odd}}}^{L_{sc}} \frac{1}{j^2} \frac{N_{0n}}{P_n}} \quad (30)$$

where B_{corr} is the single-sided bandwidth of the IF filter preceding the correlator, T_c is the averaging time of the correlator, and L_{sc} is the number of subcarrier harmonics at the BPF output. Note that if all the subcarrier harmonics pass unfiltered, then $\lim_{L_{sc} \rightarrow \infty} \sum_{\substack{j=1 \\ j: \text{odd}}}^{L_{sc}} \frac{1}{j^2} = 2\left(\frac{\pi}{4}\right)^2$.

The SNR conditioned on $\phi_c, \phi_{sc}, \phi_{sy}, \Delta\phi_{n1}$ denoted SNR'_{fsc} , is defined as before to be the square of the conditional mean of v_k divided by the conditional variance of v_k , i.e.,

$$SNR'_{fsc} = \begin{cases} \frac{2P_1 T_c}{N_{01}} C_c^2 C_{sc}^2 \left(\frac{\sum_{n=1}^L \gamma_n^2 + \sum_{n=1}^L \sum_{\substack{m=1 \\ n \neq m}}^L \gamma_n \gamma_m C_{nm}}{\sum_{n=1}^L \gamma_n} \right) & d_k = d_{k-1} \\ \frac{2P_1 T_c}{N_{01}} C_c^2 C_{sc}^2 \left(1 - \frac{|\phi_{sy}|}{\pi} \right)^2 \left(\frac{\sum_{n=1}^L \gamma_n^2 + \sum_{n=1}^L \sum_{\substack{m=1 \\ n \neq m}}^L \gamma_n \gamma_m C_{nm}}{\sum_{n=1}^L \gamma_n} \right) & d_k \neq d_{k-1} \end{cases} \quad (31)$$

Comparing (31) with the single receiver conditional SNR (12), it is clear that the term inside the large parentheses in (31) represents the ICSS than ideal gain that results due to phase misalignment between the IF signals prior to combining,

3.1 Degradation

The FSC SNR degradation is defined as the unconditional FSC SNR divided by the ideal SNR. The unconditional SNR is found by averaging (31) over the phases $\phi_c, \phi_{sc}, \phi_{sy}$, and $\Delta\phi_{n1}$ and is given as

$$SNR_{fsc} = \frac{2P_1T}{N_{01}} \overline{C_c^2} \overline{C_{sc}^2} \overline{C_{sy}^2} \left(\frac{\sum_{n=1}^L \gamma_n^2 + \sum_{n=1}^L \sum_{\substack{m=1 \\ n \neq m}}^L \gamma_n \gamma_m \overline{C_{nm}}}{\sum_{n=1}^L \gamma_n} \right) \quad (32)$$

where C_{nm} is provided in (28). The quantities $\overline{C_c^2}, \overline{C_{sc}^2}$, and $\overline{C_{sy}^2}$ are given in (15)-(17) with the modification that the loop SNRs ρ_c, ρ_{sc} , and ρ_{sy} presented in (9)-(11) are now computed using the average combined power P'/N_{0eff} which is found by averaging (26) over the phase $\Delta\phi_{n1}$, and dividing by the effective noise level in (24). Ideally, when there are no phase errors, $\overline{C_c^2} = \overline{C_{sc}^2} = \overline{C_{sy}^2} = \overline{C_{nm}} = 1$ and (32) reduce to $\left(\frac{2P_1T}{N_{01}} \sum_{n=1}^L \gamma_n \right)$. Dividing (32) by the ideal FSC SNR yields the degradation in dB, namely,

$$D_{fsc} = -10 \log_{10} \left(\overline{C_c^2} \overline{C_{sc}^2} \overline{C_{sy}^2} \left(\frac{\sum_{n=1}^L \gamma_n^2 + \sum_{n=1}^L \sum_{\substack{m=1 \\ n \neq m}}^L \gamma_n \gamma_m \overline{C_{nm}}}{(\sum_{n=1}^L \gamma_n)^2} \right) \right) \quad (33)$$

3.2 Loss

The FSC SER for an L antenna array, denoted $P_{fsc}(E)$, is defined as

$$P_{fsc}(E) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P'_{fsc}(E) \left[p_c(\phi_c) p_{sc}(\phi_{sc}) p_{sy}(\phi_{sy}) \left(\prod_{n=1}^L p_{\Delta\phi_{n1}}(\Delta\phi_{n1}) \right) \right] d\Delta\phi d\phi_{sy} d\phi_{sc} d\phi_c \quad (34)$$

where $\int_{-\infty}^{\infty}$ is the $(L-1)$ -tuple integral over the residual phases $\Delta\phi = (\Delta\phi_{21}, \dots, \Delta\phi_{(L-1)1})$. Following similar steps as in the single antenna case, the conditional SER becomes

$$P'_{fsc}(E) = \frac{1}{4} \operatorname{erfc} \left[\sqrt{\frac{E_s}{N_{01}} \left(\frac{\sum_{n=1}^L \gamma_n^2 + \sum_{n=1}^L \sum_{\substack{m=1 \\ n \neq m}}^L \gamma_n \gamma_m \overline{C_{nm}}}{(\sum_{n=1}^L \gamma_n)} \right) C_c C_{sc} \left(1 - \frac{|\phi_{sy}|}{\pi} \right)} \right]$$

$$+ \frac{E_{s1}}{4} \operatorname{erfc} \left[\sqrt{\frac{(\sum_{n=1}^L \gamma_n^2 - \frac{(\sum_{n=1}^L \gamma_n)^2}{L})}{N_{01}}} \frac{C_c C_{sc}}{(\sum_{n=1}^L \gamma_n)} \right] \quad (35)$$

where $E_{s1}/N_{01} = P_1 T/N_{01}$ is the symbol SNR at antenna 1. Ideally, when there are no phase errors, $CC = C_{sc} = (1 - \frac{|\phi_{sy}|}{\pi}) = C_{nm} = 1$ and (34) reduce to $P_{fsc}(E) = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_s L}{N_0}} \right)$ for an array of L antennas of the same size (i.e., when $\gamma_n = 1$ for all n). The symbol SNR loss for FSC is given in dB as

$$L_{fsc} = \frac{1}{4} - 20 \log \left[f^{-1}(P_{fsc}(E)) \right] \Big|_{\text{infinite loop and correlator SNR}} + 20 \log \left[f^{-1}(P_{fsc}(E)) \right] \Big|_{\text{finite loop and correlator SNR}} \quad (36)$$

after replacing $P_s(E)$ in (23) by $P_{fsc}(E)$ given in (34).

4 Numerical Results and Discussion

All the math in the previous sections reduces to two equations that give expression for the two measures of signal quality for FSC, symbol SNR degradation given by (33) and *symbol SNR* loss given by (36). After reviewing these equations, it is apparent that they are primarily a function of the various phase error variances. These, in turn, are functions of the loop bandwidths, or the correlation time, and correlation bandwidth. In other words, the quality of the combined signal is represented by two complicated expressions whose exact value varies with multiple variables. This article shows the variations of SNR degradation and SNR loss by varying the subcarrier and symbol loop bandwidths, while keeping constant the carrier loop bandwidth, correlator time, and correlation bandwidth. Furthermore, the subcarrier and symbol loop bandwidths were set to equal each other because in practice, these loops are often operated at loop bandwidths that are approximately the same.

The discussion section is divided into two parts. The first part illustrates the difference between degradation and loss. Earlier it was noted that, in general, degradation is a relative measure, whereas loss is an absolute measure of system performance. In this part it is shown

that degradation, which is much easier to compute than loss, is equal to loss at low symbol SNR, and can be used to accurately describe absolute system performance in this region. At high symbol SNR, however, it is shown that degradation is a lower bound for loss. The second part of the discussion describes the performance of the FSC technique for several different antenna combinations using the Galileo S-band mission scenario.

4.1 Degradation vs. Loss

The FSC performance for an array of two 70-m antennas when the received signal is weak is shown in Fig. 3; results for a strong signal case are shown in Fig. 4(a) for $B_c = 70$ Hz, and Fig. 4(b) for $B_c = 160$ Hz. The carrier bandwidth for the strong signal case was increased from 70 Hz to 160 Hz in which case, loss can be substantially greater than degradation as the carrier loop SNR becomes low. Inspection of these figures show that degradation and loss are equal (to within 0.01 dB) for weak signal levels, but degradation is a lower bound for loss at strong signal levels. Consequently, degradation, which in general is a relative performance measure, can be used at low symbol SNRs as an absolute measure. In other words, at weak signal levels, SNR degradation can be safely regarded as the additional SNR needed to achieve a desired SER. The benefit of using degradation instead of loss at low symbol SNRS is the significant savings in computation time.

The weak and strong signals are characterized as follows. Weak signal: $\frac{P_1}{N_{01}} = \frac{P_2}{N_{02}} = 15$ dB-Hz; $R_{sym} = \frac{1}{T} = 400$ sym/sec; strong signal: $\frac{P_1}{N_{01}} = \frac{P_2}{N_{02}} = 32$ dB-Hz; $R_{sym} = \frac{1}{T} = 400$ sym/sec. Note that the weak signal's uncombined SNR, $\frac{E_{s1}}{N_{01}} = \frac{E_{s2}}{N_{01}} = -11$ dB whereas in the strong signal case $\frac{E_{s1}}{N_{01}} = \frac{E_{s2}}{N_{01}} = 6$ dB. For an ideal system with two equal antennas, there is a 3-dB arraying gain so that the combined $\frac{E_s}{N_0}$ for the weak signal case is -8 dB which corresponds⁶ to $SER = 0.286942$, and the combined $\frac{E_s}{N_0}$ in the strong signal case is 9 dB for which the $SER = 3.4 \times 10^{-5}$. The receiver parameters for FSC in the weak signal case are

⁶ $P(b)_{ideal} = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_s}{N_0} L} \right)$ for L antennas of the same size.

assumed to be as follows: $B_c = 0.1$ Hz, B_{sc} and B_{sy} are variable, $B_{corr} = 4$ kHz, and $T_c = 120$ seconds. The following parameters apply to the strong signal case: $B_c = 70$ Hz and $B_c = 1601$ Hz, B_{sc} and B_{sy} are variable, $B_{corr} = 4$ kHz, and $T_c = 120$ seconds. Furthermore, the FSC correlator is assumed to operate on the fundamental subcarrier harmonics only, i.e., $I_{sc} = 1$ in (30).

The degradation curves for FSC are found through (33), and the loss curves are computed using (36). The loss computation is an iterative process that uses a trial-and-error method as shown by the following example. Suppose the FSC loss for $W_{sc}B_{sc} = W_{sy}B_{sy} = 2$ mHz in Fig. 3 is to be computed. First, the FSC SER is computed⁷ through (34) with $\frac{E_{s1}}{N_{01}} = -11$ dB. The resulting SER is 0.2316633 which is higher than the ideal SER of 0.286942. Consequently, a second computation of (34) is made with $\frac{E_{s1}}{N_{01}} = -11$ dB $- \Delta \frac{E_s}{N_0}$. If the resulting SER is 0.286942, then the loss is said to be $\Delta \frac{E_s}{N_0}$. If the resulting SER is greater or less than 0.286942, then (34) is recomputed with different $\Delta \frac{E_s}{N_0}$ values until the SER through (34) is equal to the ideal SER. The value of $\Delta \frac{E_s}{N_0}$ which results in (34) equaling the ideal SNR is by definition the loss. For this example, $\Delta \frac{E_s}{N_0}$ or the symbol SNR loss was found to be 0.2 dB. This method is clearly more difficult than degradation which is a single computation devoid of integrals. Nevertheless, symbol SNR loss gives the absolute performance advantage of an arraying scheme while symbol SNR degradation gives the relative performance advantage. The loop and correlator SNRs used in obtaining Fig. 3 and 4 are shown in Table 2 and 3 respectively. The FSC loop SNRs are computed using (9)-(11) using the average combined power found by averaging (26) over the residual phase and dividing by the effective noise level in (24). Moreover, the correlator SNRs for FSC were computed using (30).

⁷Note that the SER in (34) require numerical integration. An approximation to SER can be derived, however, using the moments techniques described in [10].

4.2 Galileo S-Band Mission Scenario

The FSC performance for different combinations of 70-m and 34-m antennas is discussed in this section. Since the Galileo signal is a weak signal, the performance measure used is degradation, although loss could have also been used as demonstrated in Fig. 3. As pointed out in the introduction, the IF signals in FSC are typically transmitted to central location before being combined and demodulated using a single receiver. However, since the retransmission channel is bandlimited, signal energy may be lost prior to combining. Table 4 shows the energy lost as a function of the number of subcarrier harmonics present at the central location (i.e., at the combiner input). For the Galileo scenario, four subcarrier harmonics are present at the combiner input and the energy lost is 0.22 dB.

4.2.1 Array of Two 70-m Antennas:

With that background, consider first an array of two 70-m antennas when the signal characteristics and receiver parameters are the same as those in Fig. 3 with $R_{sym} = 400$ sym/sec. FSC performance for the Galileo scenario is obtained by adding 0.22 dB to the FSC degradation shown in Fig. 3. The shifted FSC curve is plotted in Fig. 5. In addition, Fig. 5 shows results using the same parameters as in Fig. 3 but now with $R_{sym} = 200$ sym/sec (combined $\frac{P_s}{N_0} = -5.0$ dB). It is evident that as the combined symbol SNR increases (from combined $\frac{P_s}{N_0} = -8.0$ dB to -5.0 dB), the degradation correspondingly decreases, as expected. For this case, the FSC degradation ranges from 0.27 dB to 0.62 dB at $R_{sym} = 400$ sym/sec and 0.26 dB to 0.52 dB at $R_{sym} = 200$ sym/sec,

4.2.2 Array of One 70-m and One 34-m STD Antennas:

The performance of a 70-m and one 34-m STD antenna array is shown in Fig. 6(a) using the same parameters as in Fig. 3 except $\frac{P_1}{N_{01}} = -15$ dB-Hz and $\frac{P_2}{N_{02}} = 7.3$ dB-Hz, i.e., $\gamma_1 = 1$ and $\gamma_2 = 0.17$ as shown in Table 1. Fig. 6(a) also shows the results at $R_{sym} = 200$ sym/sec.

For this case, the FSC degradation ranges from 0.36 dB to 0.93 dB at $R_{sym} = 400$ sym/sec and 0.32 dB to 0.74 dB at $R_{sym} = 200$ sym/sec.

4.2.3 Array of One 70-m and Two 34-m STD Antennas:

Result for an array of one 70-m and two 34-m antennas is shown in Fig. 6(b) at the symbol rate of 200 and 400 sym/sec. For this case, the FSC degradation ranges from 0.37 dB to 0.88 dB at $R_{sym} = 400$ sym/sec and 0.34 dB to 0.72 dB at $R_{sym} = 200$ sym/sec.

4.2.4 Array of One 70-m and Three 34-m STD Antennas:

Result for an array of one 70-m and three 34-m antennas is shown in Fig. 6(c) at the symbol rate of 200 and 400 sym/sec. For this case, the FSC degradation ranges from 0.38 dB to 0.84 dB at $R_{sym} = 400$ sym/sec and 0.36 dB to 0.70 dB at $R_{sym} = 200$ sym/sec.

4.2.5 Array of Four 34-m STD Antennas:

Result for an array of four 34-m antennas is shown in Fig. 6(d) for $R_{sym} = 50$ sym/sec and $R_{sym} = 25$ sym/sec both with $B_{corr} = 400$ Hz. For this case, the FSC degradation ranges from 0.39 dB to 0.8 dB at $R_{sym} = 400$ sym/sec and 0.37 dB to 0.71 dB at $R_{sym} = 200$ sym/sec.

5 Conclusion

FSC is one of the arraying technique being considered for the Galileo spacecraft's upcoming encounter with Jupiter. Part 1 of this article described the performance of FSC technique in terms of symbol SNR degradation and symbol SNR loss. It was shown that degradation and loss were approximately equal at low values of symbol SNR, but diverge at high SNR values. Hence, missions such as the Galileo S-band mission where symbol SNR is very low, degradation rather than loss can be used to accurately describe system performance.

For the following arrays - two 70-m antennas, one 70-m and one 34-m antennas, one 70-m and two 34-m antennas, and one 70-m and three 34-m antennas - it is shown that the FSC degradation at a symbol rate of 400 sym/sec, can vary from 0.27 dB to 0.62 dB, 0.36 dB to 0.93 dB, 0.37 dB to 0.88 dB, and 0.38 dB to 0.84 dB respectively for $W_{sc}B_{sc} = W_{sy}B_{sy}$ ranging from 0.01 to 10 MHz. At the symbol rate of 200 sym/sec, on the other hand, the FSC degradation can vary from 0.26 dB to 0.52 dB, 0.32 dB to 0.74 dB, 0.34 dB to 0.72 dB, and 0.36 dB to 0.70 dB, respectively. Moreover, for an array of four 34-m antennas, the FSC degradation can vary from 0.39 to 0.8 at the symbol rate of 50 sym/sec and from 0.37 to 0.71 at the symbol rate of 25 sym/sec.

Appendix A

A.1 Derivation of (30)

The performance of the FSC correlator is derived here for the general case when the total power is divided between the data as well as the carrier. once the general correlator SNR is derived, it will be simplified for the Galileo case which operates with the carrier fully suppressed. As shown in Fig. 1 (b), the combining at IF requires both delay and phase adjustments in order to coherently add the signals. Here, perfect knowledge of the time delay is assumed and only phase compensation is required before adding the IF signals. The IF signal at antenna n [denoted by the double lines in Fig. 1 (b)] consists of an inphase (I) and quadrature (Q) component given as $r_{nI}(t)$ and $r_{nQ}(t)$ respectively

$$r_{nI}(t) = \sqrt{2P_c} \cos(\omega_c t + \theta_{cn}) - \sqrt{2P_d} d(t) \text{Sqr}(\omega_{sc} t + \theta_{scn}) \sin(\omega_c t + \theta_{cn}) + n_{nI}(t) \quad (\text{A.1})$$

$$r_{nQ}(t) = \sqrt{2P_c} \sin(\omega_c t + \theta_{cn}) + \sqrt{2P_d} d(t) \text{Sqr}(\omega_{sc} t + \theta_{scn}) \cos(\omega_c t + \theta_{cn}) + n_{nQ}(t) \quad (\text{A.2})$$

where the total power P in Watts (W) is divided between the residual carrier and data by controlling the modulation index, A . Specifically, the carrier power $P_c = P \cos^2 A$ and the data power⁸ $P_d = P \sin^2 A$. Also, $n_{nI}(t)$ and $n_{nQ}(t)$ are statistically independent with a flat one-sided PSD level equal to $N_0 W/1\text{Hz}$, and all other relevant parameters are defined in the main text. The square-wave subcarrier defined above can be expressed as follows

$$\text{Sqr}(\omega_{sc} t + \theta_{scn}) = \frac{4}{\pi} \sum_{\substack{j=1 \\ j:\text{odd}}}^{L_{sc}} \frac{\sin[j(\omega_{sc} t + \theta_{scn})]}{j} \quad (\text{A.3})$$

where L_{sc} , the number of subcarrier harmonics, is infinite. As shown in Fig. 1 (b), the IF signal from antenna 1 and n are first bandpass filtered with single-sided bandwidth B_{corr} , and then complex correlated. The output of the correlation, denoted as \tilde{z} , is a complex signal consisting of a real (I) and imaginary (Q) component, i.e.,

$$\tilde{z} = I + jQ \quad (\text{A.4})$$

⁸For the Galileo case, $\Delta = 90$ degrees so that $P_c = 0$ and $P = P_d$.

The correlator SNR at \tilde{z} , denoted $SNR'_{n1, f_{sc}}$, is defined as

$$SNR'_{n1, f_{sc}} \triangleq \frac{E(\tilde{z})E(\tilde{z}^*)}{Var(\tilde{z})} = \frac{E(\tilde{z})E(\tilde{z}^*)}{E(\tilde{z}\tilde{z}^*) - E(\tilde{z})E(\tilde{z}^*)}$$

$$\frac{E^2(I) + E^2(Q)}{Var(I) + Var(Q)} \quad (A.5)$$

where the notation $*$ represents the complex conjugate operation. Following the correlation, an averaging operation over T_c seconds is performed to reduce the noise effect. In that period, $N = 2B_{corr}T_c$ independent samples are used to reduce the variance by a factor of N . The SNR at \tilde{z} , denoted as $SNR_{n1, f_{sc}}$, is thus given by

$$SNR_{n1, f_{sc}} = SNR'_{n1, f_{sc}} N$$

$$= SNR'_{n1, f_{sc}} (2B_{corr}T_c) \quad (A.6)$$

For the general case of any I_{sc} , the correlator SNR using (A.5) can be shown to as

$$SNR'_{n1, f_{sc}} = \frac{4P_{c1}P_{cn} + 4(\frac{4}{\pi})^2 \sqrt{P_{c1}P_{cn}P_{d1}P_{dn}} (\sum_{j:odd}^{I_{sc}} (\frac{1}{j})^2) + (\frac{4}{\pi})^4 P_{d1}P_{dn} (\sum_{j:odd}^{I_{sc}} (\frac{1}{j})^2)^2}{4N_o B_{corr} [P_{c1} + P_{cn} + \frac{1}{2}(\frac{4}{\pi})^2 (P_{d1} + P_{dn}) (\sum_{j:odd}^{I_{sc}} (\frac{1}{j})^2) + B_{corr}]} \quad (A.7)$$

where B_{corr} is assumed to be sufficiently wide to pass I_{sc} subcarrier harmonic unfiltered. The correlator SNR at the output of the accumulator can now be derived by using (A.6); and, after simplification, becomes

$$SNR_{n1} = \frac{T_c \left(2 \frac{P_{c1}}{N_{01}} \frac{P_{cn}}{N_{0n}} + 2(\frac{4}{\pi})^2 \sqrt{\frac{P_{c1}}{N_{01}} \frac{P_{cn}}{N_{0n}} \frac{P_{d1}}{N_{01}} \frac{P_{dn}}{N_{0n}}} \left(\sum_{j:odd}^{I_{sc}} (\frac{1}{j})^2 \right) + \frac{1}{2}(\frac{4}{\pi})^4 \frac{P_{d1}}{N_{01}} \frac{P_{dn}}{N_{0n}} \left(\sum_{j:odd}^{I_{sc}} (\frac{1}{j})^2 \right)^2 \right)}{\frac{P_{c1}}{N_{01}} + \frac{P_{cn}}{N_{0n}} + \frac{1}{2}(\frac{4}{\pi})^2 \left(\frac{P_{d1}}{N_{01}} + \frac{P_{dn}}{N_{0n}} \right) \left(\sum_{j:odd}^{I_{sc}} (\frac{1}{j})^2 \right) + B_{corr}} \quad (A.8)$$

For $\Delta = 90$ degrees, (A.8) reduced to (30). in addition, setting $\Delta = 0$ degrees in (A.8) results in the same expression for the correlator SNR as that given in [1].

Acknowledgments

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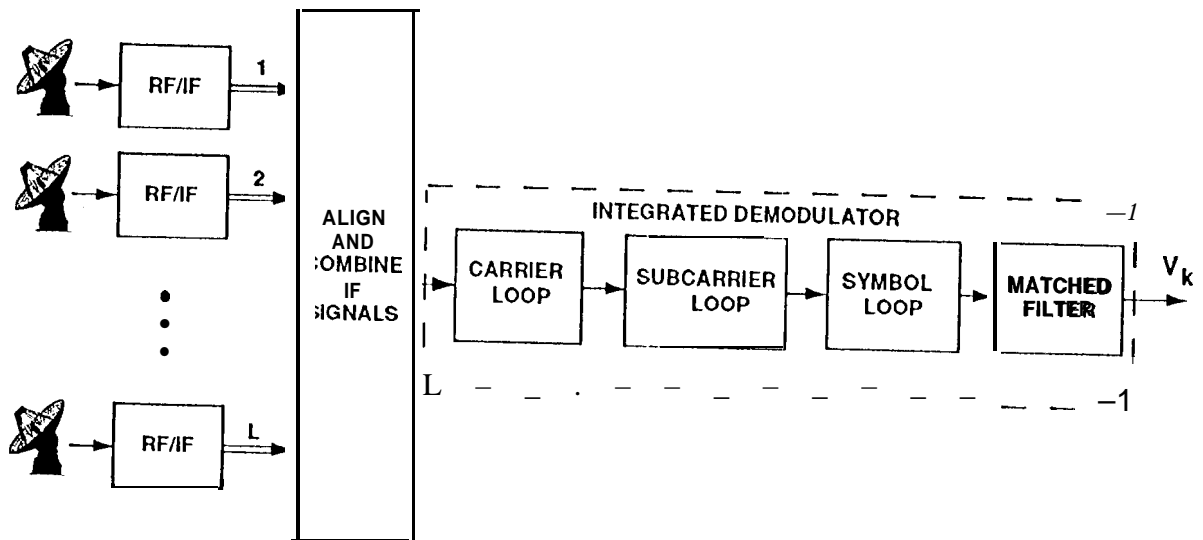


Figure 1a. Full Spectrum Combining (FSC)

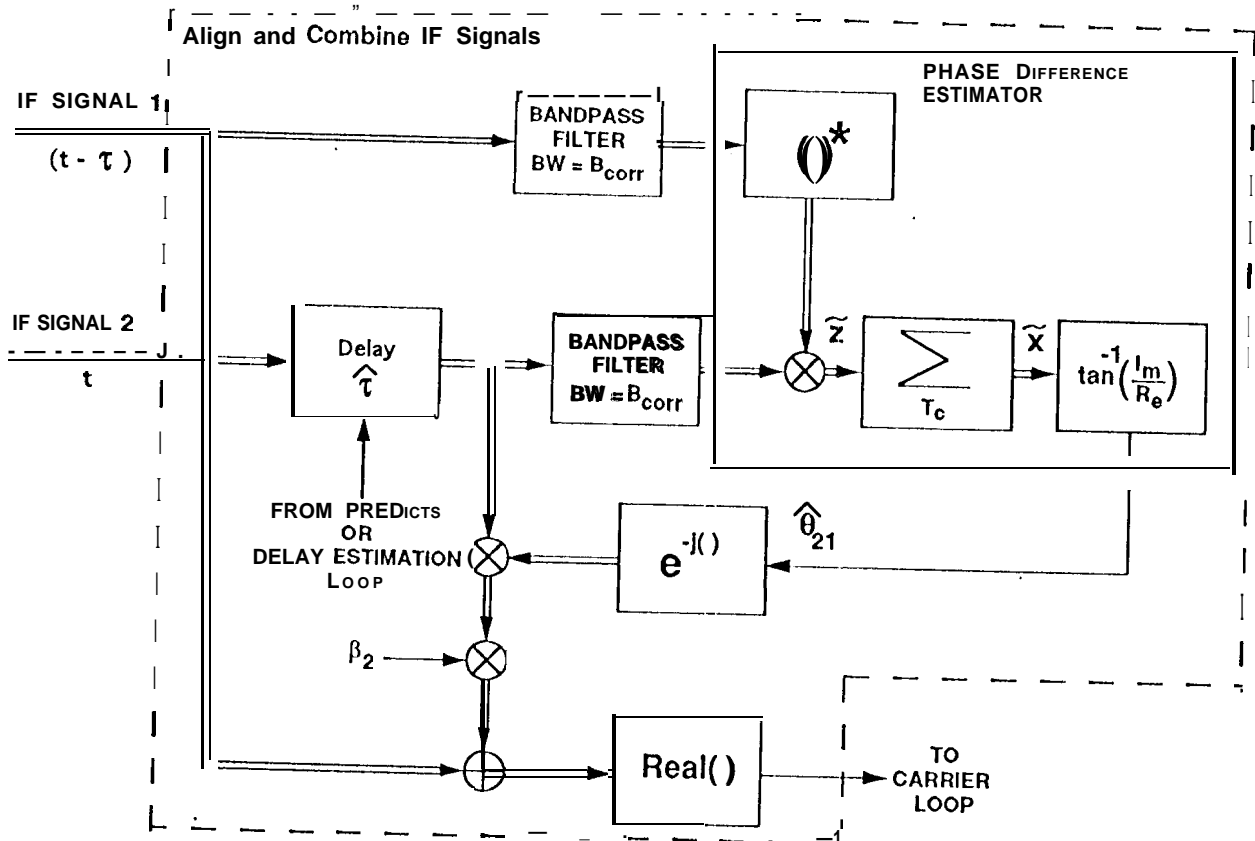


Figure 1 b. FSC Align and Combine algorithm for an array of two antennas

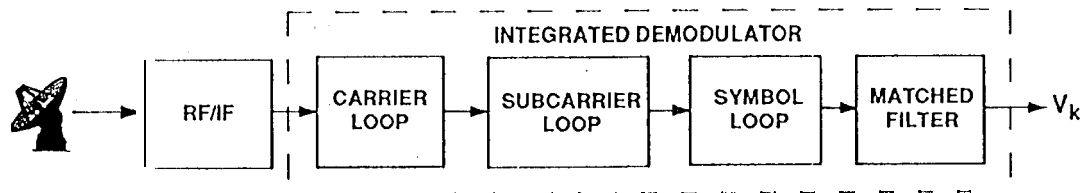


Figure 2. A general coherent receiver model.

Table 1. Gamma factors for DSN antennas

Antenna size	Frequency Band	γ_i
70 m	S-band	1.00
34 m STD	S-band	0.17
34 m HEF	S-band	0.07
70 m	X-band	1.00
34 m STD	X-band	0.13
34 m HEF	X-band	0.26

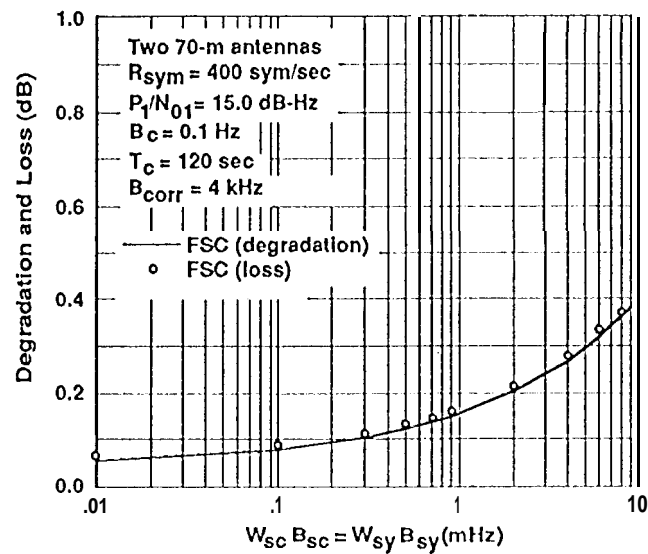


Figure 3. Degradation and loss vs subcarrier and symbol window-loop bandwidth for SER = 0.286942

Table 2. FSC loop SNRs for SER = 0.286942

$W_{\text{sc}} B_{\text{sc}} = W_{\text{sy}} B_{\text{sy}}$ (mHz)	Carrier Loop SNR (dB)	Subcarrier Loop SNR (dB)	Symbol Loop SNR (dB)	Correlator SNR (dB)
0.01	21.8	57.9	46.0	15.9
0.1	21.8	47.9	36.0	15.9
0.3	21.8	43.1	31.3	15.9
0.5	21.8	40.9	29.0	15.9
0.7	21.8	39.4	27.6	15.9
0.9	21.8	38.3	26.5	15.9
2.0	21.8	34.9	23.0	15.9
4.0	21.8	31.8	20.0	15.9
6.0	21.8	30.1	18.2	15.9
8.0	21.8	28.8	17.0	15.9
10.0	21.8	27.9	16.0	15.9

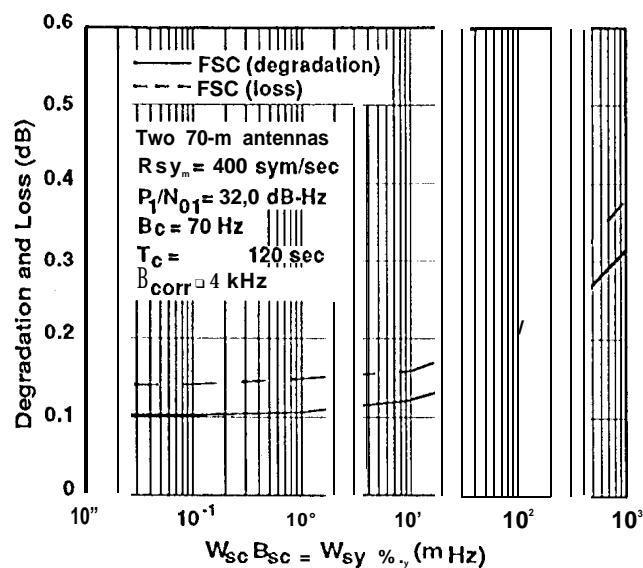


Figure 4a. Degradation and loss vs subcarrier
and symbol window-loop bandwidth for
SER = 3.4×10^{-5} at $B_C = 70$ Hz

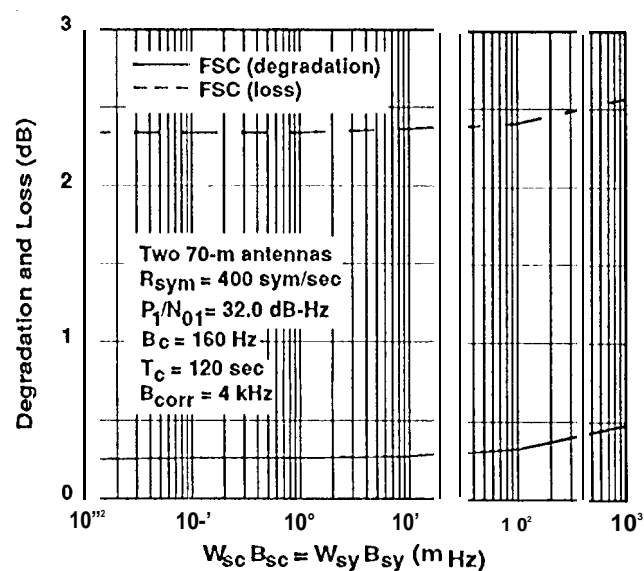


Figure 4b. Degradation and loss vs subcarrier
and symbol window-loop bandwidth for
SER = 3.4×10^{-5} at $B_C = 160$ Hz

Table 3. FSC loop SNRS for SER = 3.4×10^{-5}

$W_{sc}B_{sc} = W_{sy}B_{sy}$ (mHz)	Carrier Loop SNR (dB)		Subcarrier Loop SNR (dB)	Symbol Loop SNR (dB)	Correlator SNR (dB)
	$B_C = 160$ Hz	$B_C = 70$ Hz			
0.01	12.7	16.3	80.8	72.1	47.8
0.1	12.7	16.3	70.8	62.1	47.8
1.0	12.7	16.3	60.8	52.1	47.8
10.0	12.7	16.3	50.8	42.1	47.8
100.0	12.7	16.3	40.8	32.1	47.8
1000.0	12.7	16.3	30.8	22.1	47.8

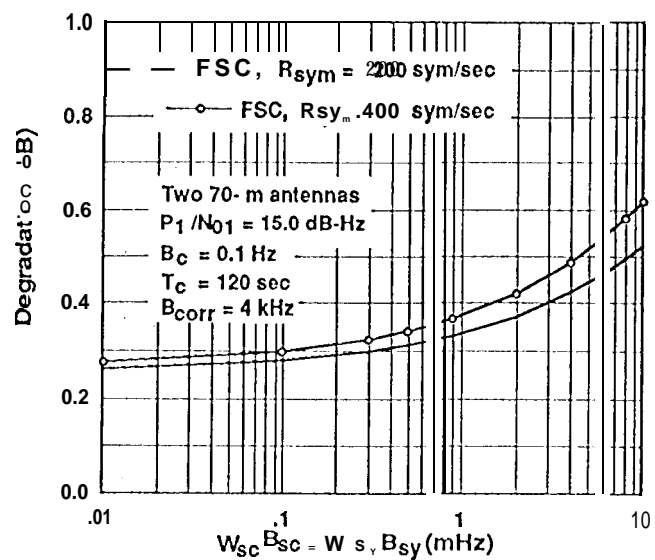


Figure 5. Practical FSC degradation vs subcarrier and symbol window-loop bandwidth

Table 4. Number of subcarrier harmonics vs loss in energy

Number of subcarrier harmonics	Loss in energy (dB)
1	0.91
2	0.45
3	0.30
4	0.22
5	0.18
6	0.15
7	0.13
8	0.11
9	0.10
10	0.07

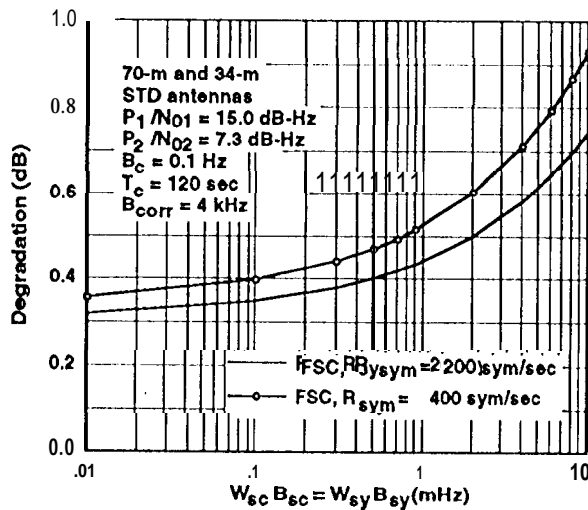


Figure 6a. Degradation for an array of one 70-m and one 34-m STD antennas

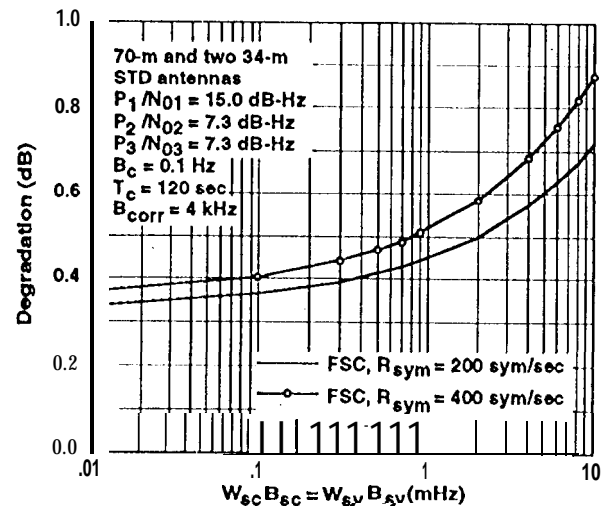


Figure 6b. Degradation for an array of one 70-m and two 34-m STD antennas

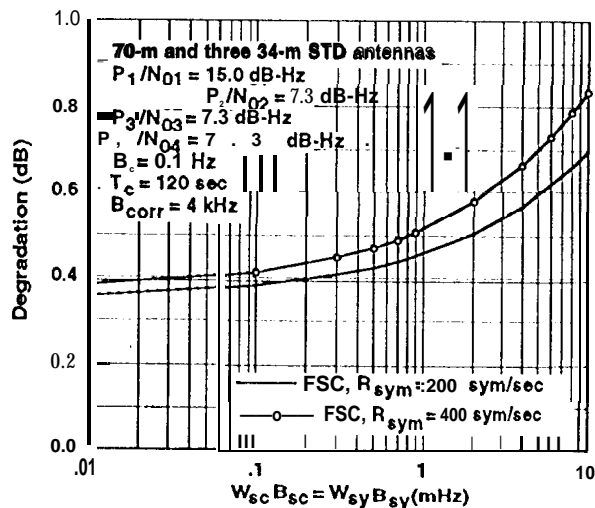


Figure 6c. Degradation for an array of one 70-m and three 34-m STD antennas

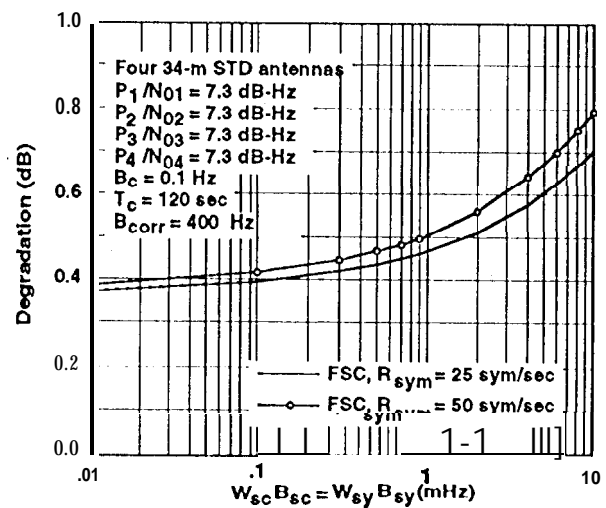


Figure 6d. Degradation for an array of four 34-m STD antennas